FLOW, HEAT AND MASS TRANSFER DUE TO INDIRECT NATURAL CONVECTION

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ABSTRACT

In the present paper we have studied the flow, heat transfer and the effect of temperature gradient on mass transfer of a thermally conducting incompressible viscous fluid which is at rest over a semi infinite uniformly heated horizontal plate facing upward due to indirect natural convection. The problem of flow, heat and mass transfer is studied by taking into consideration of Soret and Dufour effects. The governing equations of continuity, momentum, energy and concentration are transformed into non-linear ordinary differential equations and numerical solutions are obtained on the basis of boundary layer approximation by using Matlab built in solver bvp4c and presented graphically. The skin friction, Nusselt number and Sherwood number are also derived and discussed numerically.

KEYWORDS: Indirect Natural Convection, Horizontal Plate, Heat Transfer, Mass Transfer, Soret and Dufour Effect, Prandtl Number, Schmidt Number, Skin Friction, Nusselt Number, Sherwood Number

1. INTRODUCTION

If a body whose temperature is maintained at a higher value than that of the surrounding ambient fluid then density of the fluid decreases with increase in temperature resulting in buoyancy forces within it and heat transfers from heated body to the medium. In a fluid of small viscosity, a boundary layer driven directly by these buoyancy forces may be formed on a suitably oriented body. A good example of such a flow is a heated semi-infinite vertical plate, where the buoyancy force is parallel to the plate. When such a fluid flows over a horizontal heated semi-infinite plate facing upward heat is absorbed by the fluid progressively as the fluid flows over the plate; thus inducing a horizontal temperature gradient within the fluid, which in turn give rise to a favourable pressure gradient that drives the flow and then a boundary layer flow generates. If a horizontal plate is heated in such a manner that there is a temperature gradient along it then a boundary layer flow develops at the surface of the plate which in turn give rise to the induced pressure gradient that leads to indirect natural convection provided that a suitably defined Grashof number is large. Mathematically, on the surface of the horizontal plate, the temperature is everywhere T_∞, so that, as in the static field, there exists a pressure distribution having pressure gradient $\partial p / \partial y = \rho_\infty g$ where origin is taken at one of the leading edge, x is measured along the plate, y is taken normal to the plate, p is the static pressure ρ_∞ is the density of the quiescent fluid. In the boundary layer region adjacent to the plate the temperature T_w is larger than T_∞ and so the density ρ of the fluid is lower than ρ_∞. The decreased pressure gradient $\left| \partial p / \partial y \right| = \rho g < \rho_\infty g$ in the boundary layer region leads to a pressure drop in the x- direction. This reduced pressure gradient in x- direction is the origin of the indirect natural convection flow parallel to the plate and takes place at high Grashof number. It was first shown theoretically by Stewartson[1] that such an indirect natural convection flow exists

on the upper side of a horizontal plate when the temperature T_w of the plate is higher than T_∞ .

The indirect natural convection from a heated semi-infinite horizontal plate in a fluid has been investigated experimentally in recent years due to its wide range of applications in science and engineering as well as in natural circumstances such as ground water flow. Goldstein et al. [2] have investigated heat and mass transfer adjacent to horizontal plates. Lloyd and Moran [3] has studied natural convection adjacent to horizontal surface of various plane forms. Al-arabi and El-Riedy [4] have discussed natural convection heat transform an isothermal horizontal plates of different shapes. Ishiguro et al. [5] have studied heat transfer and flow instability on natural convection over upward facing horizontal surfaces. Yousef et al. [6] have investigated free convection heat transfer from upward facing isothermal horizontal surfaces. Goldstein and Lau [7] have studied laminar natural convection from a horizontal plate and the influence of plate edge extension. Sparow et al. [8] have discussed local and average natural convection Nusselt number for a uniformly heated horizontal plate. All the investigations mentioned above are experimental. Theoretical investigations reported in the literature are very few. Schneider [9] has found similarity solution for combined forced and free convection flow over a horizontal plate. Chen et al. [10] have investigated analytically the natural convection on horizontal, inclined and vertical plates with variable surface temperature. Noshadi and Schneider [11] have investigated natural convection flow far from a horizontal plate.

In this paper we have studied the flow, heat transfer and the effect of temperature gradient on mass transfer of a thermally conducting, steady, incompressible viscous fluid due to a uniformly heated horizontal plate facing upward in the boundary layer region due to indirect natural convection.
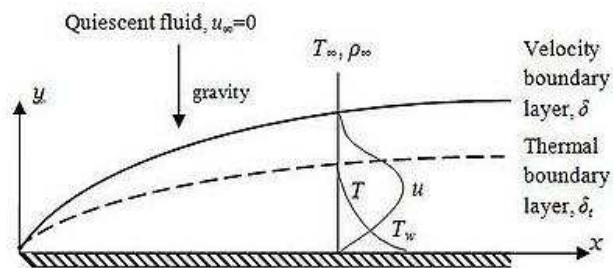
2. MATHEMATICAL MODEL

Figure 1: Schematic Representation and Coordinate System of the Problem

Consider a semi-infinite horizontal uniformly heated plate facing upward over which there is steady incompressible viscous fluid. The plate is maintained at uniform temperature T_w , while the quiescent ambient fluid is maintained at a lower temperature T_∞ . We take origin at one end of the plate, X-axis along the surface of the plate and Y-axis normal to it. The equation of continuity, momentum equations in X and Y directions, energy equation and the equation of mass transfer in the boundary layer region under Boussinesq approximation can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

<div align="right">(1)</div>

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \frac{\partial^2 u}{\partial y^2}$$

(2)

$$0 = -\frac{\partial p}{\partial y} + \overline{g}\beta_T (T - T_\infty) + \overline{g}\beta_c (C - C_\infty)$$

(3)

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{c_s c_p} \frac{\partial^2 C}{\partial y^2}$$

(4)

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m} \frac{\partial^2 T}{\partial y^2}$$

(5)

Where u, v are fluid velocity components along X-axis and Y-axis, p is the pressure, v is the kinematic viscosity, $\overline{g}$ is the acceleration due to gravity, ρ is the density of the fluid, β_T, β_C are coefficients of thermal expansion and concentration expansion, α is the thermal diffusivity, T_m is the mean fluid temperature, k_T is the thermal diffusivity ratio, c_S is the concentration susceptibility, c_p is the specific heat at constant pressure, D_m is the molecular diffusivity, T, T_W, T_∞ and C, C_W, C_∞ are temperatures and concentrations of the fluid inside the boundary layer respectively.

The boundary conditions of the problem are

$$u = 0, v = 0, T = T_W, C = C_W \qquad \text{at } y = 0$$

(6)

And $\quad u \to 0, T \to T_\infty, C \to C_\infty, p \to p_\infty \qquad \text{at } y \to \infty$

(7)

3. METHOD OF SOLUTION

We introduce the stream function $\psi(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

(8)

Where

$$\psi = \left\{x^3 v^3 \beta g (T_W - T_\infty)\right\}^{\frac{1}{5}} f(\eta), \; \eta = \frac{y}{x^{2/5}} \left\{\frac{(T_W - T_\infty)\beta g}{v^2}\right\}^{\frac{1}{5}}, p = p_\infty + \rho \left\{v\beta^2 g^2 x (T_W - T_\infty)^2\right\}^{\frac{2}{5}} g(\eta)$$

$$T = T_\infty + (T_W - T_\infty)\theta(\eta), \; C = C_\infty + (C_W - C_\infty)\phi(\eta)$$

(9)

The equation (1) of continuity is satisfied identically for the value of ψ. Substituting the transformations (8) and (9) into the equations (2) to (7) we get the following non-linear ordinary differential equations:

$$f''' + \frac{3}{5} ff'' - \frac{1}{5}(f')^2 = \frac{2}{5}(g - \eta g')$$

(10)

$$g' = \theta + N\phi \tag{11}$$

$$\theta'' + \frac{3}{5}Prf\theta' + DuPr\phi'' = 0 \tag{12}$$

$$\phi'' + \frac{3}{5}Scf\phi' + SrSc\theta'' = 0 \tag{13}$$

Together with the boundary conditions

$$f = 0, \quad f' = 0, \quad \theta = 1, \quad \phi = 1 \qquad when \qquad \eta = 0 \tag{14}$$

$$g \to 0, \quad f' \to 0, \quad \theta \to 0, \quad \phi \to 0 \qquad when \qquad \eta \to \infty \tag{15}$$

Here the primes denote differentiation with respect to η and non-dimensionless parameters are

$$Pr = \frac{v}{\alpha} = Prandtl\ No., \quad Sc = \frac{v}{D_m} = Schmidt\ No., \quad Du = \frac{D_m k_T (C_w - C_\infty)}{c_s c_p v(T_w - T_\infty)} = Dufour\ No.,$$

$$Sr = \frac{D_m k_T (T_w - T_\infty)}{T_m v(C_w - C_\infty)} = Soret\ No.\ and\ N = \frac{\beta_c (C_w - C_\infty)}{\beta_T (T_w - T_\infty)} \tag{16}$$

The parameter N measures the relative expansions of mass and thermal diffusion of the fluid.

4. NUMERICAL ANALYSIS AND DISCUSSIONS

The non-linear coupled ordinary differential equations (10) to (13) are solved numerically by using Matlab built in solver bvp4c by taking into consideration the boundary conditions equations (14) and (15). Graphical representations are shown below for various values of Pr, Du, Sr, Sc.
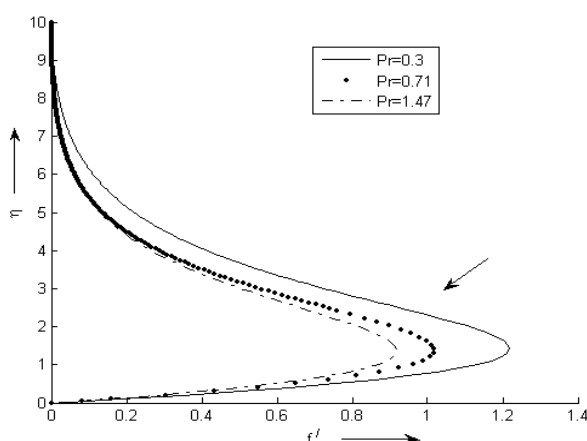

Figure 2: Velocity Distribution against η for Various Values of Pr and N=1; Sr=0.6; Sc=0.5; Du=0.6
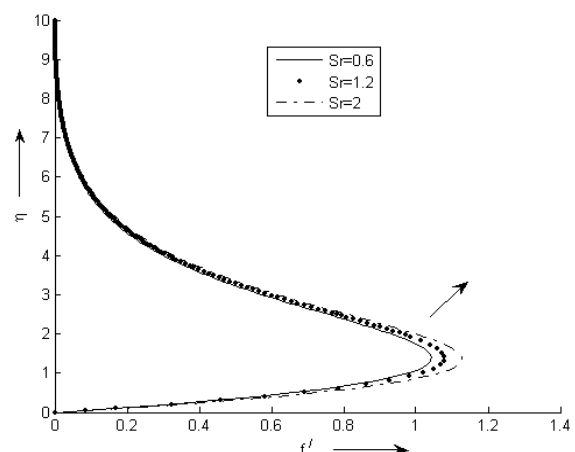
Figure 3: Velocity Distribution against η for Various Values of Sr and N=1; Pr=0.71; Sc=0.5; Du=0.8
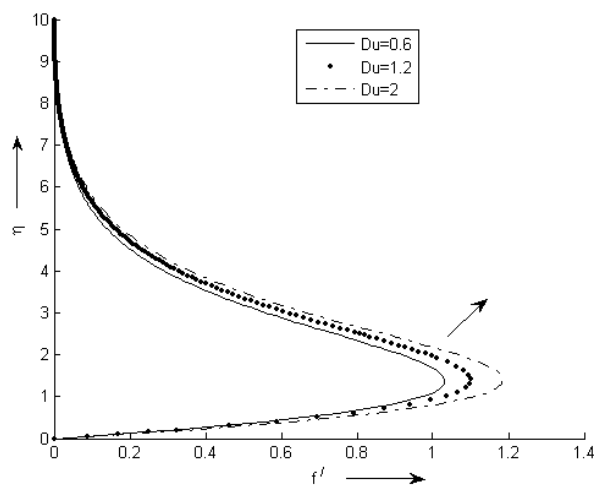
Figure 4: Velocity Distribution against η for Various Values of Du and N=1; Sr=0.8; Sc=0.5; Pr=0.71
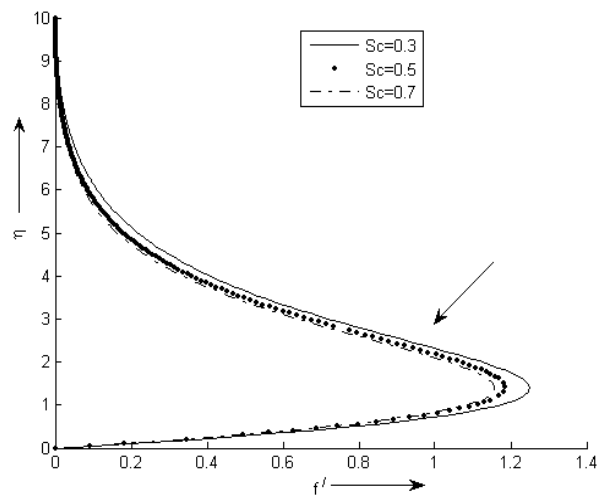
Figure 5: Velocity Distribution against η for Various Values of Sc and N=1; Sr=0.8; Du=2; Pr=0.71
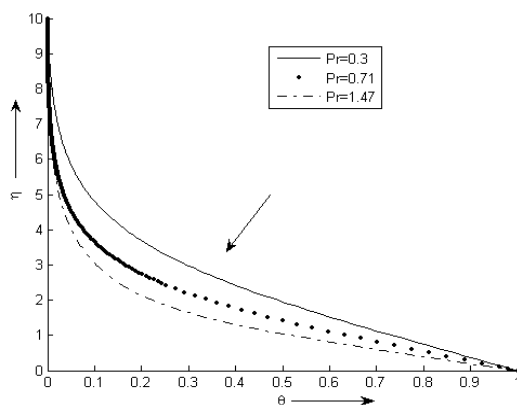
Figure 6: Temperature Distribution against η and N=1; Sr=0.6; Sc=0.5; Du=0.6
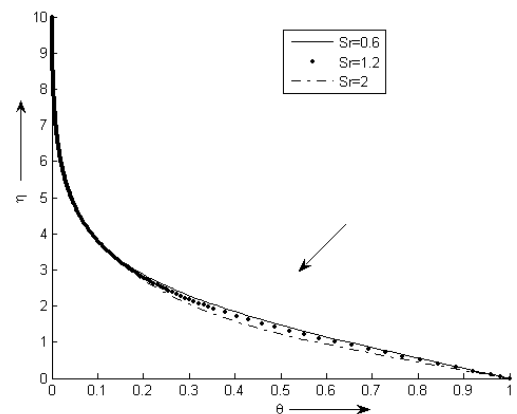
Figure 7: Temperature Distribution for Various Values of Pr against η for Various Values of Sr and N=1; Pr=0.71; Sc=0.5; Du=0.8
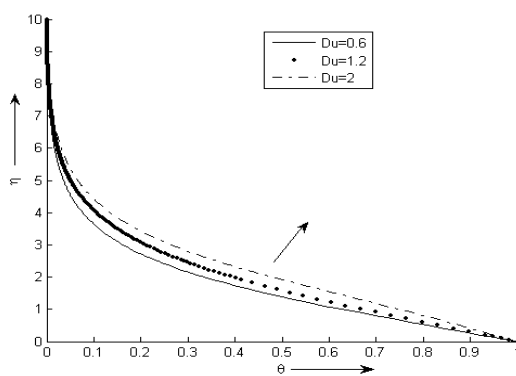
Figure 8: Temperature Distribution against η of Du and N=1; Sr=0.8; Sc=0.5; Pr=0.71
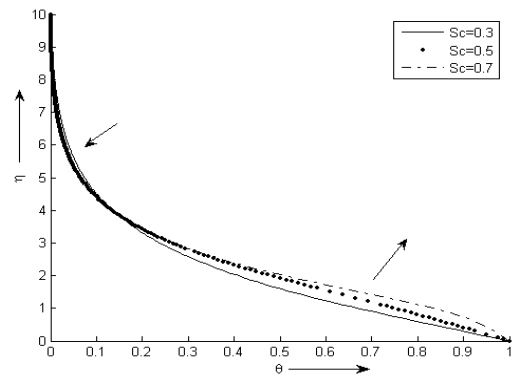
Figure 9: Temperature Distribution for Various Values against η for Various Values of Sc and N=1; Sr=0.8; Du=2; Pr=0.71
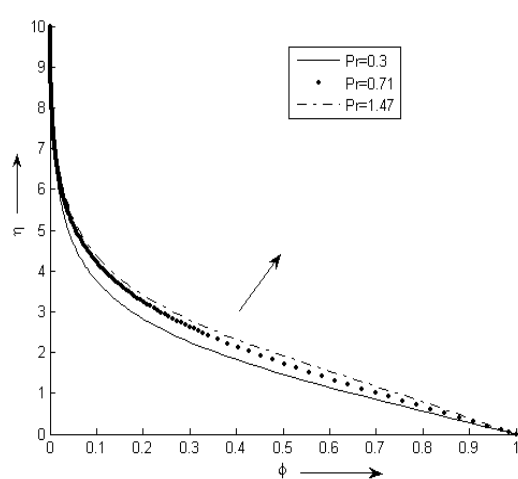
Figure10: Mass Distribution against η for Various Values of Pr and N=1; Sr=0.6; Sc=0.5; Du=0.6

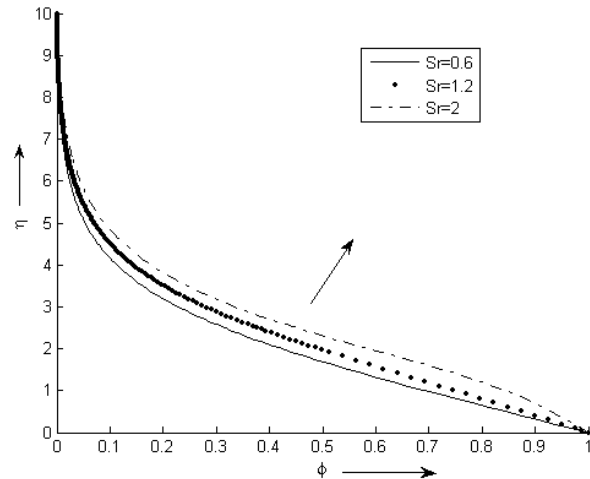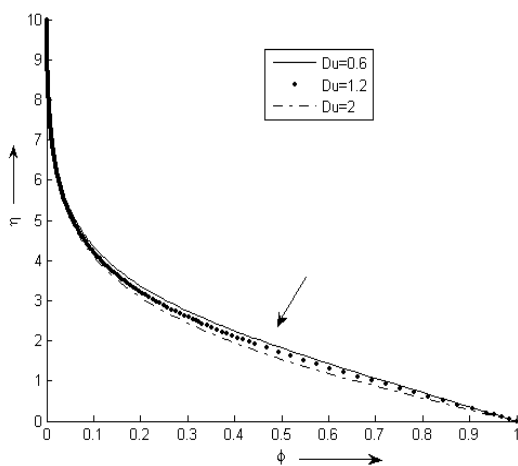**Figure 11: Mass Distribution against η for Various Values of Sr and N=1; Pr=0.71; Sc=0.5; Du=0.8**

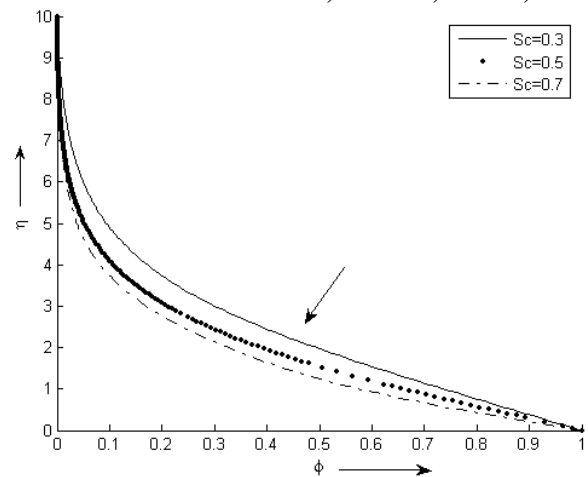**Figure 12: Mass Distribution against η for Various Values of Du and N=1; Sr=0.8; Sc=0.5; Pr=0.71**

Figure 13: Mass Distribution against η for Various Values of Sc and N=1; Sr=0.8; Du=2; Pr=0.71

It reveals from the Figure (2-5) that velocity decreases as the values of Pr and Sc increase and velocity increases as the values of Sr and Du increase. It has been noticed that the increase in velocity is more near the surface of the plate and attains its maximum value at η=1.5 approximately, and then it goes on decreasing and becomes zero at the upper edge of the boundary layer which is at η=8 approximately.

Figure (6-9) depicts that temperature of the fluid decreases as the values of Pr and Sr increase and temperature increases as the values of Du increases. Also to be noticed that temperature decreases for various values of Sc far away from the plate and becomes zero at $\eta = 4.091$ approximately and then increases near the plate. It is clear from the Figure (6-9) that Sr is less effective in case of temperature distribution.

From Figure (10-13) it is clear that concentration of mass increases as the values of Pr and Sr increase and concentration of mass decreases as the values of Du and Sc increase. Moreover it is clear that Du is less effective in mass distribution than Pr, Sr and Sc.

The effects of local skin friction, the Nusselt number and Sherwood number which have practical importance are

tabulated in Table 1. The behaviours of these parameters are self evident from the Table 1 and hence any further discussion about them seems to be redundant.

Table 1: Numerical Values of Skin Friction, Nusselt Number and Sherwood Number

N	Sc	Pr	Du	Sr	f''	$-\theta'$	$-\phi'$
1	0.5	0.3	0.6	0.6	1.8973	0.2687	0.3585
1	0.5	0.71	0.6	0.6	1.6563	0.3724	0.3016
1	0.5	1.47	0.6	0.6	1.5202	0.5068	0.2511
1	0.5	0.71	0.8	0.6	1.6852	0.3576	0.3094
1	0.5	0.71	0.8	1.2	1.7601	0.3881	0.2502
1	0.5	0.71	0.8	2	1.8564	0.4443	0.1357 0.1357 0.1357
1	0.5	0.71	0.6	0.8	1.6831	0.3805	0.2830
1	0.5	0.71	1.2	0.8	1.7662	0.3355	0.3089
1	0.5	0.71	2	0.8	1.8801	0.2451	0.3503
1	0.3	0.71	2	0.8	1.9949	0.3367	0.2640
1	0.5	0.71	2	0.8	1.8801	0.2451	0.3503
1	0.7	0.71	2	0.8	1.8300	0.1106	0.4662

CONCLUSIONS

From the above discussions it is clear that values of Prandtl number (Pr), Soret number (Sr), Dufour number (Du) and Schmidt number (Sc) play important roles in velocity, temperature and mass distributions in case of indirect natural convection on a uniformly heated horizontal plate.

ACKNOWLEDGEMENTS

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